
Calculation of Load Capacity and Water Film Thickness for Fully Grooved Water Lubricated Main Guide Bearings for Hydro Turbines

By Dr.-Ing Gary Ren, P. Eng. @ Thordon Bearings Inc. Canada

1. Introduction

Water-lubricated bearings have found widespread applications on vertical hydro-turbine as main guide bearings. The end user benefits from its simple installation, easy maintenance and environmental friendliness. Water-lubricated main guide bearings are conventionally designed with axial grooves because product water used as lubricant usually contains significant amount of abrasives with sizes mostly larger than the minimum water film thickness. Grooves are provided for the purpose on effectively flushing away the abrasives.

The design practice of water-lubricated main guide bearings is empirical based on friction data and pressure capacity from experiments (data of Stribeck-Curves). Two types of testing were developed for obtaining frictional data. One of them is to test single individual stave [1]. The data is then used to a bearing which is considered as an assembly of many individual staves. The second type of testing is to test the whole bearing [2, 8]. With using testing results, engineers and designers are able to determine the maximum allowed bearing pressure. There seems to be no theoretical method existing for determining the load capacity and water film thickness for fully grooved bearings.

There are two major difficulties on calculating the load capacity and water film thickness of fully grooved water-lubricated main guide bearings. One is the presence of grooves. And second is the surface deformation due to the relatively lower modulus of elasticity of materials which are commonly used for water-lubricated main guide bearings.

This document provides a simple, but effective method to calculate the load capacity and water film thickness based on rigid material assumption. Even though this doesn't consider the effect of elastic deformation of bearing surface, the results do provide significant amount of information about the nature of fully grooved bearings. With using this information, it is possible for engineers and designers to adjust designed parameters, such as running clearance, number of grooves, allowed bearing pressure etc to optimize the bearing performance and prolong their operational life.

2. Inclined Sliding Pad as Building Block

Due to the variety of the number and width of grooves, there is no universally suitable calculation method to be found in literatures. This paper proposed an idea that a grooved journal bearing can always be considered as an assembly of an array of simple inclined sliding pads (Figure-1). So the existing results for simple inclined sliding pads can be used as building blocks to build a calculation method for a fully grooved journal bearing.

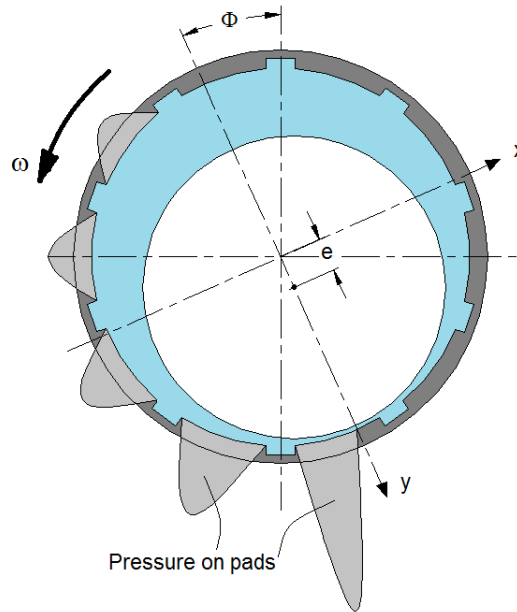


Figure-1: A grooved Bearing is an array of sliding pads (Φ = Attitude angle, e = Eccentricity)

In order to implement this idea and get a suitable accuracy for practice application, there are two (2) questions need to be answered beforehand:

- 1) What are the influencing factors which fundamentally affects the accuracy of end result for the grooved bearing if using existing information of inclined sliding pads
- 2) What information about the inclined sliding pads can be used to ensure a reasonable accuracy of the grooved bearing

The following factors may directly affect the accuracy of assembled grooved bearing:

- 1) Considering a single stave between two neighboring grooves (Figure-1), the ratio of the bearing length to the arc length of this single stave will determine whether an infinite length sliding pad to be suitable for the calculation. Stachowiak et al [3] showed that if this ratio greater than 3.0; the result of a finite length pad has almost no difference from the infinite length pad. For main guide bearings of hydro turbine, this ratio is in general large. For calculation purpose, to ensure the result is reliable, a ratio of 4.0 as minimum recommended.
- 2) Inclined sliding pads are flat surfaces. The grooved journal bearings are circular surface. The curvature of staves from the grooved bearing is different from the flat pads. However, hydro turbine main guide bearings are large and have a large number of grooves and very small running clearance, the curvature of a stave will have very little or no effect on the accuracy.
- 3) Bearing surface deformation (deflection) has been always an issue on bearing load capacity analysis. This is so called Elastohydrodynamic Lubrication problem. This paper is not dealing with this problem. Therefore, the proposed method here is theoretically only suitable to rigid bearings. However, it can also be used to deformed surface bearings in practice depending on loading and application circumferences.

Based on above reasoning, the single stave from a grooved journal bearing can be considered as the same as an infinite long inclined sliding pad. Therefore, all existing theoretical results can be used as building blocks to assemble a fully grooved journal bearing.

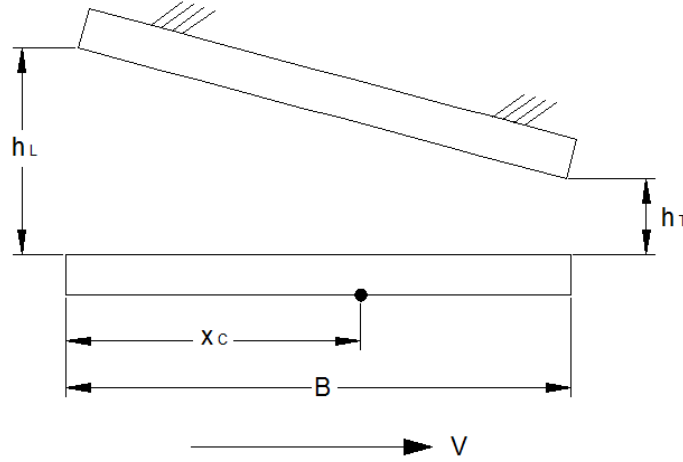


Figure-2: Infinite length inclined sliding pad

For an inclined slider, important parameters are shown on Figure-2. They are the water film thickness at leading edge " h_L ", the water film thickness at trailing edge " h_T ", the sliding velocity of pad " V " and the slider width " B " and the location of load center " x_C ". Based on infinite length theory (practically valid for $L/B \geq 3$), the characteristic dimensions of a sliding pad according to Constantinescu et al [6] are as following:

A: Load Capacity (force)

$$W = \frac{6 \cdot [(K+1) \cdot \ln K - 2 \cdot (K-1)]}{(K-1)^2 \cdot (K+1)} \cdot \frac{\mu \cdot V \cdot B^2 \cdot L}{h_T^2} \quad (1)$$

B: Location of load center (distance)

$$x_C = \frac{K \cdot (K+1) \cdot \ln \frac{1}{K} - \frac{5}{2} \cdot (K-1) - 3}{(K+1) \cdot \ln K - 2(K-1)} \cdot B \quad (2)$$

Where:

B – Width of each staves

L – Bearing length

μ - Viscosity of lubricant

V - Surface Velocity of Shaft

$K = h_L / h_T$.

According to [3], the maximum load capacity of an infinite long inclined slider can be achieved when $K = 2.2$. For this reason, an optimal bearing design shall strive to achieve $K = 2.2$ for the most loaded stave for the bearing.

3. Assembly Procedure

Figure-3 shows a fully grooved journal bearing under a steady operational condition. By given load and shaft speed, the shaft center is offset from bearing center with an eccentricity of " e ". The connecting line between bearing center and shaft center is in-line with " y "-axis of the moving coordinate system x - y . Assuming the bearing is fixed in position and the load is vertical as shown on the figure, the x - y coordinate system has an attitude angle " Φ " with respect to the loading direction. The attitude angle changes depending on load, shaft speed and groove numbers.

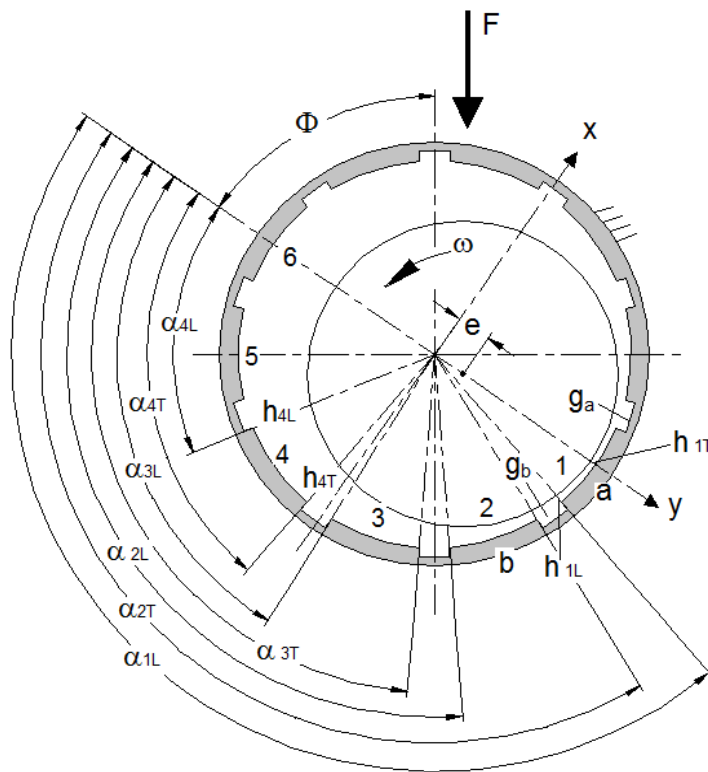


Figure-3: Grooved Bearing under Load

In shown situation, it is very interesting to realize that the y -axis divides the entire bearing into two equal halves. All staves (pads) underneath y -axis have convergent angles in shaft rotating direction and therefore are able to create hydrodynamic lifting forces. All staves (pads) above y -axis have divergent angles in shaft rotating direction and are therefore not able to create hydrodynamic lifting forces. In theory, the divergent bearing half could create even a negative pressure. However, since in practice, almost in all the cases, outside source of lubricant will be supplied to the bearing, the divergent half may not create a negative pressure, but keep the same level of pressure as supplied lubricant. It is therefore acceptable to assume the pressure on that half of bearing as zero which is corresponding to half Sommerfeld or Gumbel boundary condition. As matter of fact, only half number of total pads will take the load which is the ones underneath the y -axis in the sense of Figure-3.

Updated on May 14, 2014

Since the bearing is assumed to be fixed on earth, all angles (α_{iL}, α_{iT} $i = 1, 2, 3 \dots N/2$) defining the positions of grooves will change with attitude angle which is an unknown parameter. This difficulty can be overcome by a trick which assigns a "floating number" on each stave (pad) underneath the y-axis. As a rule, no matter how the attitude angle to change, it is always the first convergent stave underneath y-axis at the smallest water film location is called number "1". Other staves are enumerated clockwise with number 2, 3, 4 etc in sequence. Within the context of this paper, the term pad and stave means the same.

In Figure-3, there could be 3 situations possible. First situation is that the y-axis falls into groove " g_a ". In this case, the pad numbered with "1" is an entire pad. Second situation is that the y-axis falls into groove " g_b ". In this case, thanks to the usage of floating numbering system, the current pad #2 will become pad #1. This means these two situations are same from point of view of calculation. The third situation just as illustrated in Figure-3 is that the y-axis falls in between two grooves. In this case a single pad is divided into two parts by the y-axis. The part underneath of y-axis will contribute hydrodynamic support to load capacity while the part above the y-axis has no contribution to bearing load capacity.

By using the floating numbering system, the angles defining the positions of grooves with respect to the moving x-y coordinate system is easy to calculate.

Assuming the y-axis falls into a pad (pad number 1 on Figure-3); the angle of trailing edge of pad "1" is always equal " π ". So the angle of leading edge will be " π " less an angle corresponding to the convergent port of pad which can be assumed as " $\lambda \cdot \Delta\alpha$ " with λ as convergent portion of the pad.

So location angles of grooves with respect to the x-y coordinate system are calculated as below:

$$\alpha_{iT} = \pi \quad (3a)$$

$$\alpha_{iL} = \pi - \lambda \cdot \Delta\alpha \quad (3b)$$

In case that the y-axis falls into a groove, the trailing and leading edge angle of pad number "1" is calculated by

$$\alpha_{iT} = \pi - 0.5 \cdot \Delta\gamma \quad (3c)$$

$$\alpha_{iL} = \alpha_{iT} - \Delta\alpha \quad (3d)$$

Other angles are simply following the first ones and calculated as below:

$$\alpha_{iT} = \alpha_{i-1L} - \Delta\gamma \quad (4a)$$

$$\alpha_{iL} = \alpha_{iT} - \Delta\alpha \quad (i=2, 3, 4 \dots N/2) \quad (4b)$$

Where:

λ - The convergent portion of pad #1 on hydrodynamic side ($\lambda = 0$ to 1). The physical meaning of parameter " λ " is the percentage of pad #1 surface which shares the load.

Updated on May 14, 2014

N - Total number of grooves

$\Delta\alpha$ - Angle corresponding to the arc of one pad

$\Delta\gamma$ - Angle corresponding to one groove

α_{iT} - Groove location angle at trailing edge

α_{iL} - Groove location angle at leading edge

The film thickness will be with known angles from equation (3) and (4) according to Hamrock [4]

$$h_{iT} = c \cdot (1 + \varepsilon \cdot \cos \alpha_{iT}) \quad (5a)$$

$$h_{iL} = c \cdot (1 + \varepsilon \cdot \cos \alpha_{iL}) \quad (i=1, 2, 3 \dots N/2) \quad (5b)$$

Where:

$\varepsilon = e/c$ Eccentricity ratio

c = Bearing radial clearance

h_{iT} - Water film thickness at trailing edge

h_{iL} - Water film thickness at leading edge

According to definition, the minimum water film thickness of bearing is

$$h_{\min} = h_{iT} = c \cdot (1 + \varepsilon \cdot \cos \alpha_{iT}) \quad (6)$$

Now, we need to define a new parameter which is the ratio of water film thickness at leading edge to trailing edge.

$$K_i = \frac{h_{iL}}{h_{iT}} \quad (i=1, 2, 3 \dots N/2) \quad (7)$$

Applying result of equation (5a) and (5b) to (1) and (2), the load capacity and the location of force center for each individual pad can be calculated. As usual, the first pad is always an exception because only a partial of pad will take load:

$$W_1 = \frac{6 \cdot [(K_1 + 1) \cdot \ln K_1 - 2 \cdot (K_1 - 1)]}{(K_1 - 1)^2 \cdot (K_1 + 1)} \cdot \frac{\mu \cdot V \cdot B^2 \cdot L \cdot \lambda^2}{h_{iT}^2} \quad (8a)$$

$$W_i = \frac{6 \cdot [(K_i + 1) \cdot \ln K_i - 2 \cdot (K_i - 1)]}{(K_i - 1)^2 \cdot (K_i + 1)} \cdot \frac{\mu \cdot V \cdot B^2 \cdot L}{h_{iT}^2} \quad (8b)$$

$$x_{iC} = \frac{K_i \cdot (K_i + 1) \cdot \ln \frac{1}{K_i} - \frac{5}{2} \cdot (K_i - 1) - 3}{(K_i + 1) \cdot \ln K_i - 2(K_i - 1)} \cdot B \quad (9)$$

The projected components in x-axis direction and y-axis direction are further calculated with help of equation (8a, 8b) and (9).

$$F_{ix} = W_i \cdot \sin\{\pi - [\xi_i \cdot \alpha_{iT} + (1 - \xi_i) \cdot \alpha_{iL}]\} \quad (10a)$$

$$F_{iy} = W_i \cdot \cos\{\pi - [\xi_i \cdot \alpha_{iT} + (1 - \xi_i) \cdot \alpha_{iL}]\} \quad (10b)$$

Where: $\xi_i = x_{iC} / B$ ($i = 1, 2, 3 \dots N/2$)

The total supporting force contributed by all pads underneath y-axis is then the sum of all components above:

$$F_x = \sum_{i=1}^{N/2} F_{ix} \quad (11a)$$

$$F_y = \sum_{i=1}^{N/2} F_{iy} \quad (11b)$$

The resultant force is then

$$F = \sqrt{F_x^2 + F_y^2} \quad (12)$$

The attitude angle:

$$\Phi = \tan^{-1} \frac{F_x}{F_y} \quad (13)$$

In all equations above there are two parameters unknown and interdependent which is the eccentricity ratio $\varepsilon = e/c$ and parameter λ in equation (3b). Here iteration technique is required and also manual involvement may be necessary. By first run of calculation, a λ can be arbitrarily assumed to start the calculation. Then by reversely using equation (12) the eccentricity ratio can be calculated. And attitude angle will be determined by equation (13). If the calculated attitude angle is different from the one assumed, adjust parameter λ and re-run the calculation. Repeating the same procedure until the right attitude angle obtained.

Example #1:

One main guide bearing for a vertical hydro turbine has following given information:

- Shaft diameter: $d = 600$ mm
- Bearing length: $L = 600$ mm
- Diameter running clearance: $cd = 0.2$ mm
- Shaft Speed: $n = 200$ rpm
- Given Bearing Pressure: $P_n = 0.3$ MPa

Other parameter after running Thordon Sizing Program is

- Number of Grooves: $N = 12$
- Width of groove: $W_g = 10 \text{ mm}$
- Width of Pad: $W_p = 147 \text{ mm}$
- Pad arc angle: $\Delta\alpha = 28.1^\circ$
- Groove arc angle: $\Delta\gamma = 1.91^\circ$
- Load "F" direction to: Center of one pad

Search for load distribution among the staves (pads) of bearing. Following the procedure as described before, the calculation result is shown in table-1.

Table-1: Result of example #1 (Nominal Pressure = 0.3 MPa)

Pad number	Force on Pad (N)	Shared by %	Pressure on pad MPa
1	9875	8.19 by 0.4 pad	0.28
2	87990	78.57	0.998
3	11240	10.04	0.127
4	2104	1.88	0.024
5	586	0.523	0.007
6	185	0.165	0.002

It is very interesting to notice that the most loaded pad #2 alone shared 78.57% of total load. This pad is subject to a pressure of 0.998 MPa. Pad #1 has only 40% ($\lambda=0.4$) of surface area taking its share. But the pressure on it is still lower than its next neighboring pad #2. The pads 3 shares approximately the same amount of load as pad#1, but lower than pad#2. These 3 pads take 97% of total load. Another fact is that the total sum of individual forces of each pad together is not the same of applied vertical load "F". It is the sum of vertical projections of these forces to balance the vertical load "F". Table-1 indicates that the pad #2 is loaded most. The average pressure is 0.998 MPa. Figure-4 shows the pressure over individual pad for all pads.

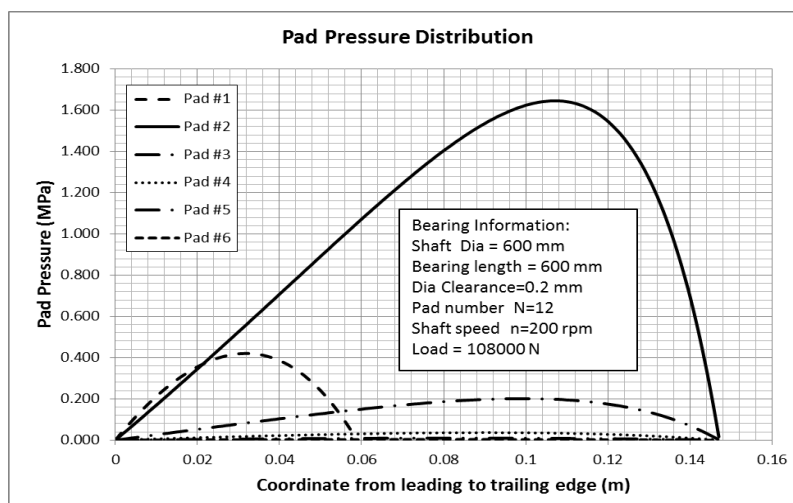


Figure-4: Pressure distribution over each individual pad

Other result of this example is:

Minimum water film thickness:	$h_{\min} = 9.6 \mu\text{m}$
Eccentricity ratio:	$\varepsilon = 0.904$
Calculated attitude angle:	$\Phi = 27.2 \text{ degree}$
Required parameter:	$\lambda = 0.4$

Using the same input data, the Childs finite length bearing theory provided a higher water film thickness and smaller eccentricity ratio:

Minimum film thickness according to Childs [9,10]:	$h_{\min} = 36.5 \mu\text{m}$
Eccentricity ratio:	$\varepsilon = 0.636$
Calculated attitude angle:	$\Phi = 49.3 \text{ degree}$

It is clear to see bearing grooves significantly reduces minimum water film thickness and load capacity. The minimum film thickness is about 4 times lower than the non-grooved bearing. The prediction without considering groove effect would be over optimistic in terms of lubrication film thickness and load capacity.

4.0 Influence of Load Direction

In example #1, the attitude angle $\Phi = 27.2^\circ$, the loading taking portion of pad #1 $\lambda = 0.4$. According to Figure-3, there shall be following relation existing:

$$\Phi = \lambda \cdot \Delta\alpha + \Delta\gamma + \chi \cdot \Delta\alpha \quad (14)$$

The first term on right hand side is the angle covering pad #1. The second term is angle covering one groove. The third term is the angle covering pad #2. The portion of pad #2 is

$$\chi = \frac{\Phi - \lambda \cdot \Delta\alpha - \Delta\gamma}{\Delta\alpha} \quad (15)$$

Applying given data, we have

$$\chi = 0.504$$

This simply means that the bearing load “F” is pointing to the center of pad #2. Now we have two parameters which are λ & χ . Parameter λ defines the spot of minimum film thickness to one pad. Parameter χ defines the load direction to another pad. In example #1, $\chi = 0.504$ is not a coincidence. It was chosen on purpose when starting calculation. In general, the attitude angle is a function of many variables:

$$\Phi = \Phi(\Delta\alpha, \Delta\gamma, \chi, \lambda, V, F) \quad (16)$$

Equation (14) is a special case of equation (16) and only true for example #1. For instance, the same bearing, if the shaft speed is high and load is light, the attitude angle would be large to cover several

grooves and pads depending how wide the grooves and pads. Each case needs specific attention depending on bearing load, speed and groove numbers. If the load direction is known, such as in cases of a bearing for horizontal installation in that the load is the weight of rotor and the direction is clearly vertical. Designer can set the parameter " χ " to a fixed value (not necessarily 0.5) which determines the load direction to the bottom pad after installation.

For vertical shaft applications such as hydro turbine main guide bearings, where the load direction is uncertain, designer can run the calculation by set parameter λ to a fix value. Later on this document confirms that the effect of parameter λ on minimum film thickness is minor so that it can be arbitrarily chosen a value between 0 and 1 without dramatically affecting result.

Example #2:

Based on the same input data as example #1, by changing the nominal bearing pressure and calculate the load capacity. For a vertical hydro turbine, the direction of radial load cannot be determined by design stage. In this example, it continues using $\chi = 0.5$, namely the load is pointing to the center of pad #2. Table -2 is the given information about bearing load. Shaft speed is kept as $n = 200$ rpm.

Table -2: Given Information for example #2

	unit	Bearing Load					
Bearing Load	kN	36	72	108	144	180	216
Nominal Pressure	MPa	0.1	0.2	0.3	0.4	0.5	0.6
Parameter λ	N/A	0.653	0.491	0.4	0.332	0.274	0.219

Figure-5 shows the calculated force on each individual pad. Figure-6 is the average pressure on each pad. The average pressure is the force divided by the surface area of pad. Except for pad #1, the surface area is equal 600×147 for this example. The surface area of pad #1 is $600 \times 147 \times \lambda$. Figure-7 is the calculated minimum water film thickness and Figure-8 is the attitude angle.

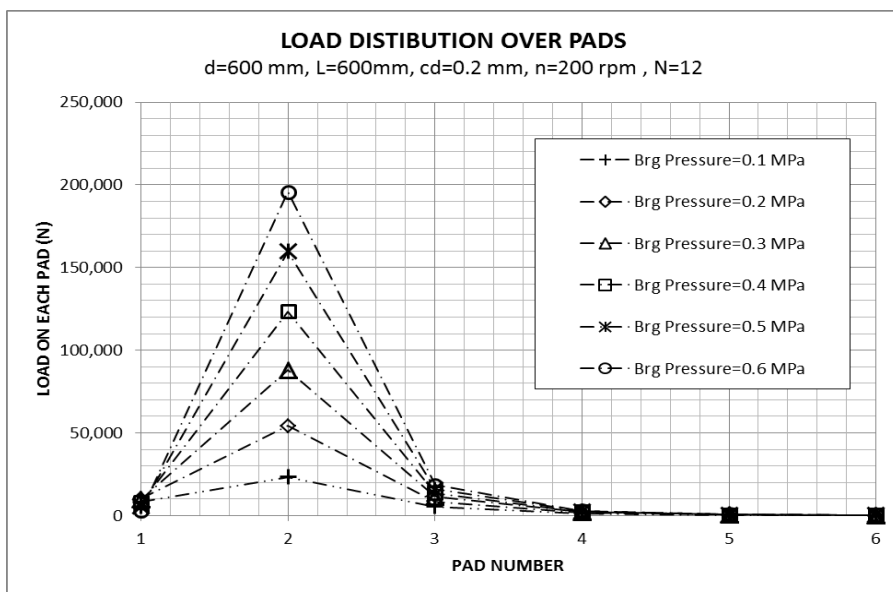


Figure-5: Load distributed to each pad

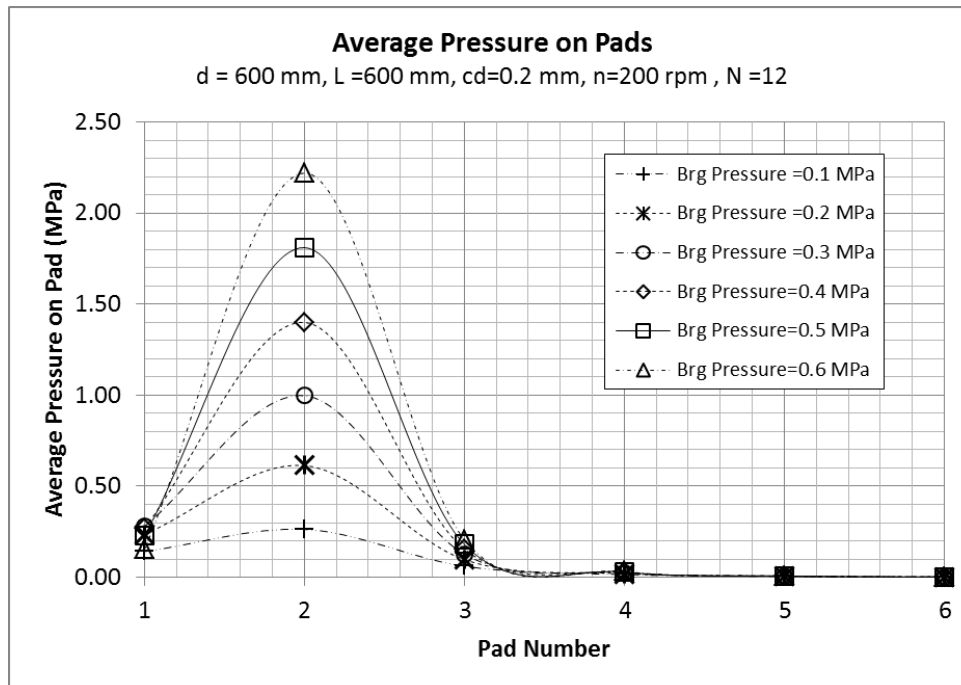


Figure-6: Average Pressure on each individual pad

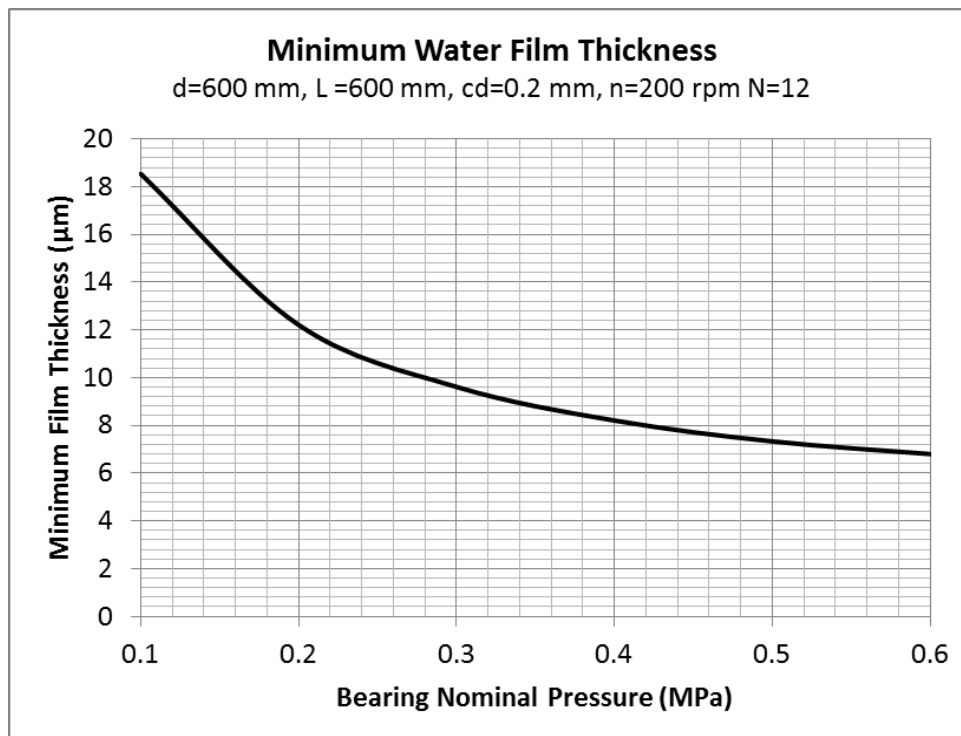


Figure -7: The minimum water film thickness of entire bearing

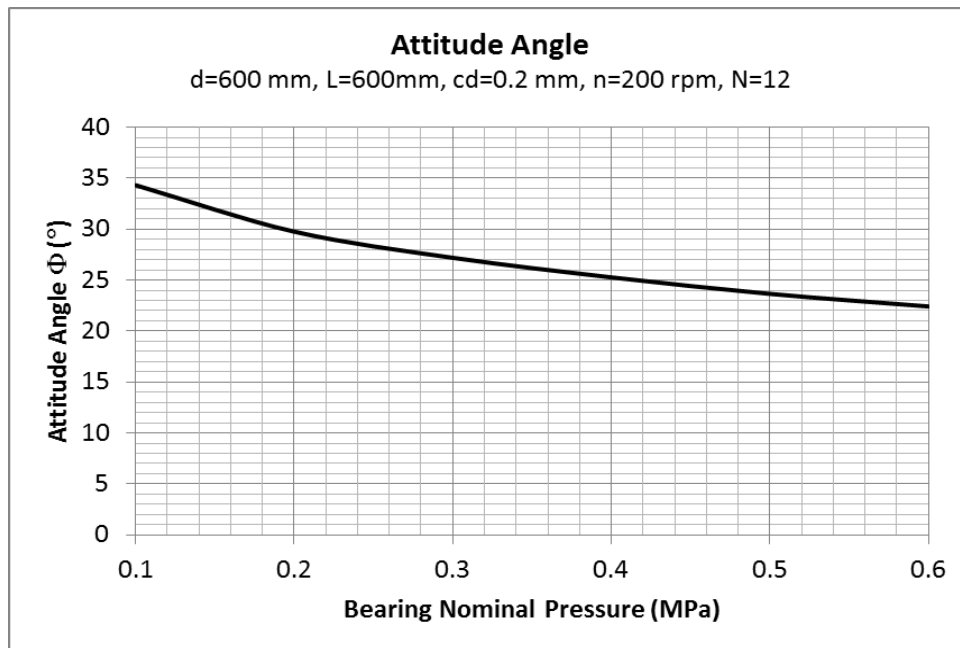


Figure -8: The attitude angle of shaft relative to bearing

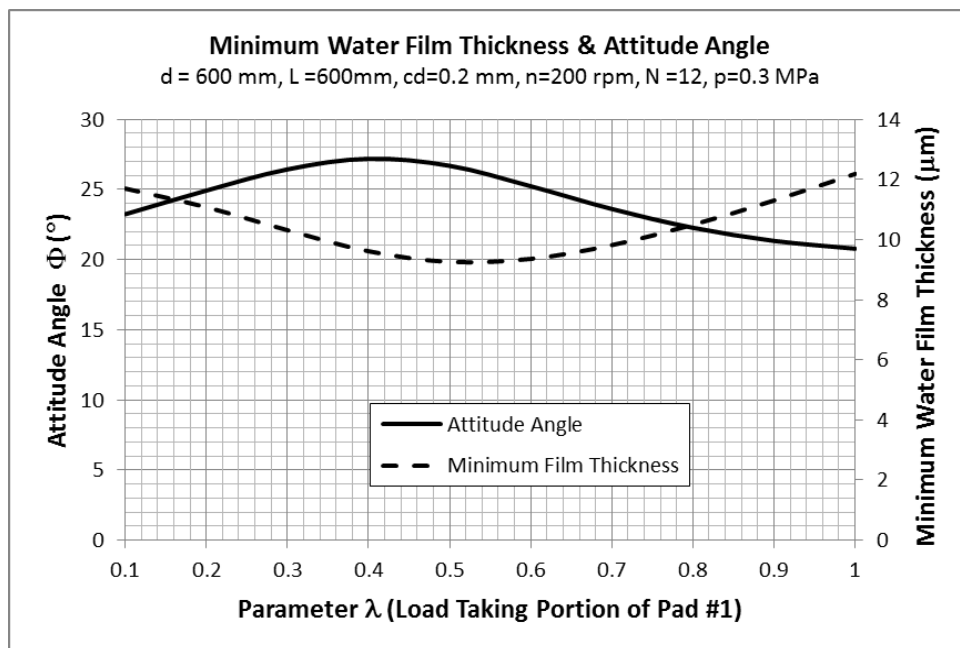


Figure -9: Influence of Parameter λ on Water Film Thickness

In example #1 and #2, the load was assumed pointing to the center of pad #2. This is $\chi=0.5$. If this constraint is removed, then the parameter λ can vary from 0 to 1.0. Figure-9 shows influence of parameter λ on the minimum water film thickness and attitude angle. From practice application point of view, the influence of parameter " λ " on minimum water film thickness is insignificant. As mentioned before, parameter " λ " determines the position of minimum film thickness towards one pad. And

parameter " χ " determines the position of load direction towards another pad. If load direction is known, such as for horizontal installation, one can fix " χ " to a preset number and calculate parameter " λ " over relationship (16) by iteration. In case of the load direction is unknown, one can set a prefix number for parameter " λ " and calculate parameter " χ " over relationship (16) by iteration. For a vertical hydro turbine main guide bearing the load direction is uncertain at design stage and parameter λ can be arbitrarily taken as 0.5 which presents the worst case scenario with the least water film thickness.

5.0 Case study of a practice main guide bearing for a vertical turbine

This section shows one of real world turbine main guide bearing. The given design parameters were as following:

Shaft diameter:	1350 mm
Housing inside diameter:	1430 mm
Housing length:	1000 mm
Designed running clearance:	0.3 mm
Groove number:	18
Groove width:	10 mm
Normal load:	503 kN
Operation Speed:	50 rpm

Table-3 summarized results for two most loaded pads.

Table -3: Load on the most loaded pads and water film thickness (Pad numbering refers to Figure-3)

	$\lambda = 0.5$				$\lambda = 1.0$			
Load (kN)	Pad #1		Pad #2		Pad #1		Pad #2	
	110.5		349.5		393.5		98.69	
Percent %	21.5		68.1		76.5		19.2	
Water film Thickness at (μm)	Leading Edge	Trailing Edge	Leading Edge	Trailing Edge	Leading Edge	Trailing Edge	Leading Edge	Trailing Edge
	8.4	6.4	25.1	8.5	16.3	8.5	40.3	17.0

For a vertical turbine, the direction of load is undetermined. Therefore, calculation chose two extreme cases. $\lambda = 0.5$ means the minimum film spot at 50% width of pad #1. $\lambda = 1$ means the minimum film spot at trailing edge of pad #1.

A dimensionless film parameter is general used to determine the lubrication regime [4]. This parameter is defined as

$$\Lambda = \frac{h_m}{\sqrt{R_{q,S}^2 + R_{q,B}^2}} \quad (17)$$

$R_{q,S}$ - RMS surface finish of shaft

$R_{q,B}$ - RMS surface finish of bearing.

$$R_q = 1.11 \cdot R_a$$

R_a - Roughness average of surface finish of mating members.

For main guide bearings of hydro turbines, the recommended shaft finish $R_a = 0.8 \mu\text{m}$. The root mean square is $R_{q,s} \approx 0.9 \mu\text{m}$. The bearing surface finish after break-in procedure is ranged from $R_a = 0.3$ to $0.9 \mu\text{m}$. So $R_{q,B} \approx 1.0 \mu\text{m}$, the film parameter would be

$$\Lambda = \frac{h_m}{\sqrt{0.9^2 + 1.0^2}} = \frac{h_m}{1.35}$$

Hamrock [4] indicates if the film parameter is greater than 5, bearing shall work in hydrodynamic regime. For main guide bearings, this means the minimum required film thickness would be:

$$h_m = 1.35 \times 5 \approx 7 \mu\text{m}$$

According to table-3, the water film thickness at trailing edge of pad #1 in case of $\lambda=0.5$ seems to be less than required film thickness. However, the calculation was based on rigid bearing theory. For deformed bearing, the water film thickness would be improved. Overall, above bearing should work fairly well hydrodynamically.

6.0 Conclusion

This document provides a calculation procedure to determine loading capacity and minimum water film thickness for fully grooved water lubricated main guide bearings for hydro turbines. To ensure the reliability of result certain conditions are applied and must be fulfilled. These are

1. The ratio of bearing length to the width of stave must be greater than 4.0. The higher, the better.
2. Bearing pressure shall be relatively low so that the surface deformation effect doesn't overwhelmingly change the result. For this reason, it doesn't recommend to apply this procedure to main guide bearings made of Thordon Composite.

The calculation procedure is able to provide load distribution on individual staves. The calculated water film thickness can be used to evaluate the suitability of bearing to an application. With using this technique, it is possible for engineers and designers to adjust design parameters to optimize bearing performance. The method can also be applied to identify the weak spots of existing designs and as a tool for failure analysis and decision making on an improved design.

References:

- [1] Roy L. Orndorff, Jr, Water-Lubricated Rubber Bearings, History and New Developments, Naval Engineers Journal, November 1985, pp 39 - 52
- [2] T. L. Daugherty, Frictional Characteristics of Water-Lubricated Compliant Surface Stave Bearings, ASLE, Transactions, Volume 24, 3, pp 293 – 301
- [3] Gwidon W. Stachowiak, Andrew W. Batchelor, Engineering Tribology, Butterworth Heinemann
- [4] Bernard J. Hanrock, Fundamentals of Fluid Film Lubrication, McGraw-Hill Inc. 1994
- [5] Michael M. Khonsari, Applied Tribology, 2th Edition, John Wiley & Sons, Ltd, 2008
- [6] Virgiliu Niculae Constantinescu, Sliding Bearings, Allerton Press, Inc / New York, 1985
- [7] Amdreas Z. Szeri, Fluid Film Lubrication, Theory and Design, Cambridge University Press, First Edition 1998
- [8] Roy L. Orndorff, Jr. New UHMWPE/Rubber Bearing Alloy, Journal of Tribology, ASME JANUARY 2000, Vol. 122. pp 367 – 373
- [9] Mircea Rades: Dynamics of Machinery, II, Editura Printech, 2009, p99-102
- [10] D. Childs, H. Moes: H. Van Leeuwen: Journal Bearing Impedance Descriptions for Rotordynamic applications, Transactions of ASME, p198, April, 1977